SOUND WAVES

IIT-JEE Syllabus

1. *Stationary Waves*
2. *Vibration of air column*
3. *Resonance, Beats*
4. *Speed of sound*

Total No. of questions in **Sound waves** are:

Solved examples.....19
Exercise # 1 25
Exercise # 2 35
Exercise # 3 25
Exercise # 4 12
Exercise # 5 10
Total No. of questions.....**126**

*** Students are advised to solve the questions of exercises in the same sequence or as directed by the faculty members.

Index : Preparing your own list of Important/Difficult Questions

Instruction to fill

(A) Write down the Question Number you are unable to solve in **column A** below, by Pen.

(B) After discussing the Questions written in **column A** with faculties, strike off them in the manner so that you can see at the time of Revision also, to solve these questions again.

(C) Write down the Question Number you feel are important or good in the **column B**.

EXERCISE NO.	COLUMN :A	COLUMN :B
	Questions i am unable to solve in first attempt	Good/Important questions
1		
2		
3		
4		
5		

BRAINS EDUCATION

Advantages

1. It is advised to the students that they should prepare a question bank for the revision as it is very difficult to solve all the questions at the time of revision.

2. Using above index you can prepare and maintain the questions for your revision.

BRAINS
EDUCATION

KEY CONCEPT

1. Nature of sound waves

A vibrating source in a medium produces sound. These sound waves are longitudinal in nature and are divided into three categories.

(a) Audible waves : Sound waves with frequency between 20 Hz and 20,000 Hz are called audible waves.

(b) Infrasonic waves : Sound waves whose frequency lie below 20 Hz are called infrasonic waves.

(c) Ultrasonic waves : Sound waves whose frequency is greater than 20000 Hz are called ultrasonic waves.

Now we shall discuss about the propagation of sound waves :

Sound waves propagate in the form of compression and rarefaction of medium. Due to propagation of these waves, pressure varies periodically in sinusoidal fashion. If P_0 is the normal pressure then excess pressure varies which is given by

$$P_f - P_0 = \Delta P_0 \sin \omega t$$

or $P = P_0 \sin \omega t$

Where P_0 is amplitude of pressure variation and P is excess pressure i.e. above or below the normal value.

2. Equation of sound waves

A sound wave is explained either by longitudinal displacement suffered by the particles of the medium or by excess pressure generated due to compression and rarefaction.

(a) If P(0,t) is known and sound wave is propagating in +X direction

$P(x,t) = P(0, t - x/v)$

$= P_0 \sin \omega(t - x/v)$

(b) If P(0,t) is known i.e. $P(0,t) = P_0 \sin \omega t$ and sound wave is propagating in –X direction

$P(x,t) = P(0, t + x/v)$

$= P_0 \sin\omega(t + x/v)$

(c) If P(x, 0) is known i.e. $P(x,0) = P_0 \sin kx$ and sound wave is propagating in +X direction

$$P(x,t) = P(x - vt, 0)$$
$$= P_0 \sin k(x - v t)$$
$$P = P_0 \sin (kx - \omega t)$$

(d) If P(x,0) is known i.e. $P(x,0) = P_0 \sin kx$ and sound wave in propagating in –X direction

$$P(x,t) = P(x + vt, 0)$$
$$= P_0 \sin k(x + vt)$$
$$P = P_0 \sin (kx + \omega t)$$

These are the equation of sound wave in terms of pressure. While obtaining them in terms of displacement of medium particles, we have to proceed in the same manner as that being done in transverse wave. So we have

$$s = s_0 \sin (\omega t - kx)$$

3. Relation between pressure and displacement equation of sound wave

Consider a sound wave going in X direction in a medium. At time t, the position of the particle suffering a displacement s is given by

$s = s_0 \sin (\omega t - kx)$ .....(1)

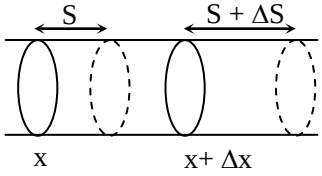
If we consider the element confined within x and x + Δx having area of cross-section A than the volume of this element is V = A.Δx. Now as the wave passes from one end to another it causes an increase in volume given by

$dV = (s + \Delta s - s)A = \Delta s.A$

$\therefore$ Volume strain $= \dfrac{\Delta V}{V} = \dfrac{A\Delta s}{A\Delta x} = \dfrac{\Delta s}{\Delta x} = \dfrac{\partial s}{\partial x}$

So $\dfrac{\partial s}{\partial x} = \dfrac{dV}{V} = -s_0 k \cos(\omega t - kx)$ {from eqn. (1)}

If B is the bulk modulus of material then,



$$B = -\frac{P}{dV/V}$$

$$\Rightarrow \quad \frac{dV}{V} = -\frac{P}{B}$$

$$-\frac{P}{B} = -s_0 k \cos(\omega t - kx)$$

$$P = s_0 kB \cos(\omega t - kx)$$

$$P = P_0 \cos(\omega t - kx)$$

Where $P_0 = s_0 k B = \frac{2\pi s_0 B}{\lambda}$

The pressure wave is 90º out of phase w.r. t displacement wave i.e. displacement is maximum when pressure is minimum and vice versa.

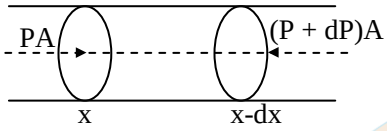
4. Velocity of sound waves

Assume a sound wave going in +X direction and is represented by equation

$s = s_0 \sin(\omega t - kx)$ ...(i)

The pressure variation is given by

$P = P_0 \cos(\omega t - kx)$ ...(ii)



From the figure it is clear that resultant force on the element at time t is

$F = PA - (P + dP)A = - dP.A$

From eqn. (ii)

$dP = -P_0 \sin(\omega t - kx).(-k)\, dx$

$= kP_0 \sin(\omega t - kx)dx$

So Net force $F = -k P_0 \sin(\omega t - kx)\, A.dx$

If ρ is density of fluid then mass of the element is ρAdx. So acceleration of the particle is

$$\alpha = \frac{F}{\rho.A.dx} = \frac{-kP_0 \sin(\omega t - kx)}{\rho} \quad ...(iii)$$

But acceleration of the medium particle is given by $\alpha = \frac{\partial^2 s}{\partial t^2} = -\omega^2 s_0 \sin(\omega t - kx)$ ...(iv)

Comparing (iii) and (iv)

$$-s_0\omega^2 \sin(\omega t - kx) = -\frac{kP_0}{\rho} \sin(\omega t - kx)$$

$$s_0\omega^2 = \frac{kP_0}{\rho}$$

$$\omega = \sqrt{\frac{kP_0}{\rho s_0}}$$

Put $P_0 = 2\pi s_0 B/\lambda$

$$\omega = \sqrt{\frac{k}{\rho s_0}.\frac{2\pi s_0 B}{\lambda}} = \sqrt{\frac{k.k.B}{\rho}} = k\sqrt{\frac{B}{\rho}}$$

or $2\pi n = \frac{2\pi}{\lambda}\sqrt{\frac{B}{\rho}}$

$$n = \frac{1}{\lambda}\sqrt{\frac{B}{\rho}}$$

Thus $v = n\lambda = \sqrt{\frac{B}{\rho}}$

4.1 FOR SOLIDS, speed of sound wave is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

Where Y is young's modulus of the solid and is density of solid.

4.2 FOR LIQUIDS $v = \sqrt{\frac{B}{\rho}}$

4.3 FOR GASES

According to Newton, when a sound wave propagates in a gas, the temperature variations are negligible i.e the temperature is constant. Hence propagation of sound is an isothermal process. So by Boyle's law

$PV = \text{Constant}$ or $P\Delta V + V\Delta P = 0$

$$B = -\frac{\Delta P}{\Delta V/V} = P$$

Hence speed of sound in gas is given by

$$v = \sqrt{\frac{P}{\rho}}$$

Now if $P = 1.01 \times 10^5$ N/m^2 and = 1.29 kg/m^3

So speed of sound in air at C is

$$= \sqrt{\frac{1.01\times10^5}{1.29}} = 280 \text{ m/s}$$

But by experiment, speed of sound in air is 331 m/s. Hence the above formula was modified by Laplace. He assumed that temperature of the gas does not remain constant and assume sound propagation to be adiabatic process.

$PV^\gamma = \text{constant}$

Taking logarithm

$\log P + \gamma \log V = \text{constant}$

or $\frac{\Delta P}{P} + \gamma\frac{\Delta V}{V} = 0$

$$B = \frac{-\Delta P}{\Delta V/V} = \gamma P$$

So speed of sound in gas $V = \sqrt{\frac{\gamma P}{\rho}}$

Note:(1)In the case of gases

$\rho = \frac{\text{Mass}}{\text{Volume}}$ and $PV = nRT$

where $n = \frac{\text{Mass}}{\text{Mol.wt.M}}$

$v = \sqrt{\frac{\gamma RT}{M}}$

(2) From kinetic theory of gases,

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{v}{v_{rms}} = \sqrt{\frac{\gamma}{3}} \Rightarrow v_{rms} = v.\sqrt{\frac{3}{\gamma}}$

(3) Average speed of the molecules of a gas is

$v_{av} = 0.92\ v_{rms}$

(4) Most probable speed is given by

$v_{mp} = 0.816\ v_{rms}$

(5) $\gamma = C_P/C_V$ and has values and for mono, dia and polyatomic gases respectively.

5. Factors affecting speed of sound in gas

The speed of sound in a gas depends on the temperature, relative humidity and pressure.

(a) Effect of temperature : Velocity of sound in gas is given by

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Hence velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

If v_0 is speed of sound at 0°C and v_t at t°C then

$v_0 = \sqrt{\frac{\gamma R}{M}.(273+0)}$

$v_t = \sqrt{\frac{\gamma R}{M}.(273+t)}$

So $\frac{v_t}{v_0} = \sqrt{\frac{273+t}{273}} = \left[1+\frac{t}{273}\right]^{1/2}$

$v_t = v_0\left[1+\frac{t}{546}\right]$

(b) Effect of Relative Humidity : With increase in humidity, density decreases and hence velocity of sound increases. In rainy seasons, sound travels faster as compared to the summer.

(c) Effect of Pressure :

The speed of sound in a gas is given by

$v = \sqrt{\frac{\gamma P}{\rho}}$

From the formula it looks as if speed changes with pressure. But we have

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

So at constant temperature, if P changes then also changes such that P/ρ is constant. Hence pressure dose not have any effect on velocity of sound till temperature remains constant.

6. Intensity of sound waves

When the wave travels in a medium it carries energy with it. The intensity of a sound wave is defined as the average energy transmitted per unit normal area per unit second i.e. power per unit area.

Consider a sound wave travelling along x direction. We have

$s = s_0 \sin(\omega t - kx)$

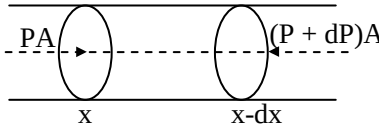
and $P = P_0 \cos(\omega t - kx)$

Where $P_0 = \frac{2\pi s_0 B}{\lambda}$

Now if we assume a cross-section of area A on which a force pA is exerted so that points of application move with a speed . Hence power (W) transmitted

$W = \text{Force} \times \text{Velocity}$

$= PA \times \frac{\partial s}{\partial t}$



But intensity is Power/Area i.e. $I = P.\frac{\partial s}{\partial t}$

$I(t) = P_0 \cos(\omega t - kx).\ s_0\omega \cos(\omega t - kx)$

$= P_0\ s_0\ \omega \cos^2(\omega t - kx)$

$$= \langle I \rangle = \frac{\int_0^T I(t)}{T}$$

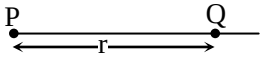
So intensity I is equal to average power transmitted over unit area i.e

$$I = \frac{2\pi B s_0^2 \omega}{2\lambda}$$

$$I = \frac{P_0^2 \lambda \omega}{4\pi B}$$

Hence Intensity is proportional to the square of pressure amplitude.

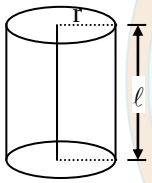
(i) If the source is a point source, then with increase in distance from the source, the intensity decreases i.e



$$I = \frac{W}{4\pi r^2}$$

or $$I \propto \frac{1}{r^2}$$

(ii) If a sound source is a line source emitting sound than intensity at a distance r is given by



$$I = \frac{W}{2\pi r \ell}$$

6.1 LOUDNESS OF A SOUND:

It depends on the intensity and frequency of sound. But human ear responds to sound intensities over a wide range. Hence a logarithmic scale of intensity i.e sound level was given by

$$SL = 10 \log_{10} \frac{I}{I_0}$$

Where I is intensity of sound and I_0 is threshold of human ear.

6.2 RELATIVE SOUND LEVEL:

It is the comparison between two different sounds which is given by

$$SL_1 - SL_2 = 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0}$$

$$SL_1 - SL_2 = 10 \text{ Log } \frac{I_1}{I_2}$$

6.3 PITCH :

It is the characteristic of sound which differentiates between the shrill or grave sound. It depends on the frequency of sound.

6.4 QUALITY OF SOUND :

It is the characteristics which differentiates between the sounds of same intensity and frequency.

Note :

1. Energy associated with the sound wave is given by

$$E = \frac{1}{2} \rho \omega^2 A.dx$$

2. Power i.e Rate of energy is given by

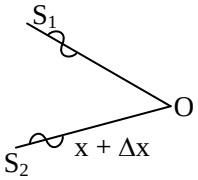
$$P = \frac{1}{2} \rho v \omega^2 A$$

3. Intensity = Energy density × velocity

7. Interference of sound waves

According to principle of Superposition, when two or more sound waves pass through the same point then the resultant disturbance is the sum of disturbance produced by individual sound waves. At some points intensity is maximum where as at some other points it is minimum making the condition of constructive interference and destructive interference.

Consider two waves of same frequency and same amplitude reaching at point p from two sources S_1 and S_2. Let p_1 and p_2 be the pressure amplitude of wave from s_1 and s_2 so that pressure change at O is given by



$P_1 = p_1 \sin(kx - \omega t)$

$P_2 = p_2 \sin[(kx - \omega t) + \phi]$

where $\phi = \frac{2\pi \Delta x}{\lambda}$ is the phase difference between two waves. Hence resultant wave at P is,

$P = P_1 + P_2$

$= p_1 \sin(kx - \omega t) + p_2 \sin(kx - \omega t + \phi)$

$= p_1 \sin(kx - \omega t) + p_2 \sin(kx - \omega t) \cos\phi$

$+ p_2 \sin\phi \cos(kx - \omega t)$

$= (p_1 + p_2 \cos\phi) \sin(kx - \omega t)$

$+ p_2 \sin\phi \cos(kx - \omega t)$

Let $p_1 + p_2 \cos\phi = p_0 \cos\theta$ ...(1)

and $p_2 \sin\phi = p_0 \sin\theta$ ...(2)

$P = p_0 \cos\theta \sin(kx - \omega t) + p_0 \sin\theta \cos(kx - \omega t)$

$P = p_0 \sin(kx - \omega t + \theta)$

Squaring and adding eqn (1) and (2)

$(p_0 \cos\theta)^2 + (p_0 \sin\theta)^2$

$= (p_1 + p_2 \cos\phi)^2 + (p_2 \sin\phi)^2$

$p_0^2 = p_1^2 + p_2^2 \cos^2\phi + 2p_1 p_2 \cos\phi + p_2^2 \sin^2\phi$

$p_0^2 = p_1^2 + p_2^2 + 2 p_1 p_2 \cos\phi$ ...(3)

$p_0 = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\phi}$

Dividing eqn (2) by (1) we have

$$\frac{p_0 \sin\theta}{p_0 \cos\theta} = \frac{p_2 \sin\phi}{p_1 + p_2 \cos\phi}$$

$$\tan\theta = \frac{p_2 \sin\phi}{p_1 + p_2 \cos\phi}$$

(i) For constructive interference

$\Delta\phi = 2n\pi$ and $Dx = n\lambda$

$\Rightarrow$ $p = p_1 + p_2$

(ii) For destructive interference

$\Delta\phi = (2n + 1)$ and $\Delta x = \left(n + \frac{1}{2}\right)\lambda$

$\Rightarrow$ $p = |p_1 - p_2|$

(a) If $p_1 = p_2$ and $\Delta x = \lambda/2$

Resultant $p = 0$ i.e. no sound

(b) If $\Delta x = \lambda$ i.e $\Delta\phi = 2\pi$ and $p_1 = p_2$

$p_0^2 = p_1^2 + p_1^2 + 2 p_1 p_1 \cos 2\pi$

{from eqn(3)}

$= 4 p_1^2$

$p_0 = 2p_1$

i.e amplitude of resultant wave becomes two times and hence resultant intensity of sound becomes four times the intensity due to individual source.

8. Reflection of sound waves

The two conditions under which reflection of sound takes place are :

(a) Discontinuity in the medium i.e wave is continuous at the boundary. When a sound wave is reflected from a rigid boundary, the particle are not able to vibrate at the boundary. So a reflected wave is generated. This wave interferes with the incident wave so that net displacement is zero giving rise to maximum variation in pressure. Hence phase of reflected wave remains same i.e a compression pulse is reflected as compression pulse and vice versa.

(b) **Low pressure region:** When the sound wave travels in open tube, it gets reflected from the open end by which force is exerted on the particles due to outside air. This force is of small magnitude and so the particles vibrate with increased amplitude. So pressure remains at average value by which the phase change of π occurs. Hence compression pulse is reflected as rarefaction pulse and vice versa.

(c) Echo is an example of reflection of sound from a distant object. If there is a sound reflector at a distance d from the source then time taken to hear the echo is 2d/v. Where d is distance of object to source.

Note - Multiple or successive echoes will be produced where there are two or more reflectors.

9. Stationary wave

When two exactly identical sound wave propagate in opposite direction superimpose then resultant wave is called stationary wave.

$p_A = p_0 \sin(kx - \omega t)$

$p_B = p_0 \sin(kx + \omega t)$

By principle of superposition,

$p = p_A + p_B$

$= p_0 \sin(kx - \omega t) + p_0 \sin(kx + \omega t)$

$= 2 p_0 \sin kx \cos \omega t$

Thus amplitude of oscillation of the resultant wave is $2p_0 \sin kx$ which depends on parent wave.

The results obtained from above equation are –

(i) Different points oscillate with different amplitude.

(ii) Few points oscillate with maximum amplitude (pressure amplitude). The point with maximum amplitude (pressure variation) are called pressure antinodes. In terms of displacement they are referred as displacement node because at these points, displacement amplitude is zero.

(iii) Few points with minimum amplitude (pressure amplitude). The point with minimum amplitude (pressure variation) are called pressure nodes. In terms of displacement, they are referred as displacement antinodes because at these displacement amplitude is maximum.

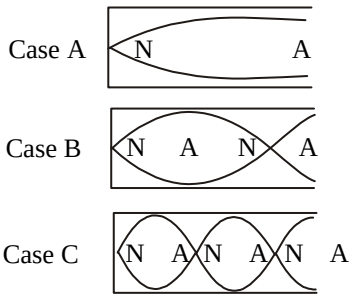
(iv) Separation between two consecutive nodes and antinodes is $\lambda/2$.

10. Vibration of air columns

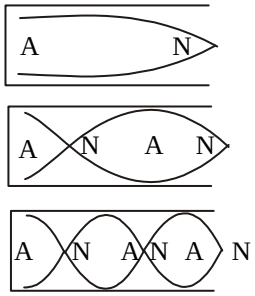
When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition; standing waves are produced. These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

10.1VIBRATION OF AIR COLUMN IN CLOSED ORGAN PIPE :

The tube which is closed at one end and open at the other end is called closed organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the waves reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrate where as at open end there is an antinode because particles have greatest freedom to vibrate. Hence on blowing air at the open end, the air column vibrates forming antinode at free end and node at closed end. If ℓ is length of pipe and λ is wavelength



In term of displacement



In terms of pressure

Case A →

$\ell = \lambda/4$ $\lambda = 4\ell$

$n_1 = v/\lambda = v/4\ell$ fundamental frequency

Case B →

$\ell = 3\lambda/4$ $\lambda = 4\ell/3$

$n_2 = v/\lambda = 3v/4\ell$ First overtone frequency

Case C →

$\ell = 5\lambda/4$ $\lambda = 4\ell/5$

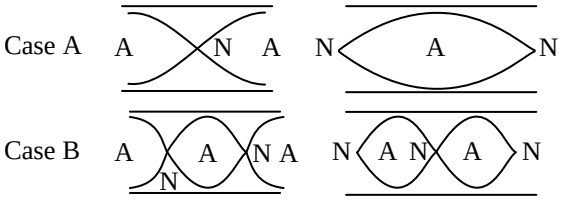
$n_3 = v/\lambda = 5v/4\ell$ Second overtone frequency

Hence frequencies of the fundamental tone and the overtones are given by the relation

$n_1 : n_2 : n_3$ .....= 1 : 3 : 5.....

10.2VIBRATION OF AIR COLUMNS IN OPEN ORGAN PIPE :

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travel towards the other end from where it is reflected back. As the wave reaches the first end, it is reflected again. Thus two longitudinal waves travel in opposite direction to superpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and $\frac{\lambda_1}{\lambda}$ be the wavelength.



In terms of displacement In term of pressure

Case A →

$\ell = \lambda/2$ $\lambda = 2\ell$

v is velocity of sound in organ pipe

$n_1 = v/\lambda = v/2\ell$ Fundamental frequency

Case B →

$\ell = 2\lambda/2 \qquad \lambda = 2\ell/2$

Thus frequency $n_2 = v/\lambda$

First overtone frequency

$$n_2 = 2v/2\ell$$

Hence frequencies of fundamental tone and overtones are given by the relation

$n_1 : n_2 : n_3 ..... = 1 : 2 : 3 .....$

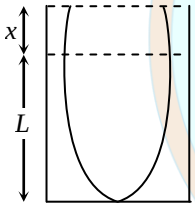
Note.

(1) A rod clamped at one end or a string fixed at one end is similar to vibration of closed end organ pipe.

(2) A rods clamped in the middle is similar to the vibration of open end organ pipe.

(3) If an open pipe is half submerged in water, it becomes a closed organ pipe of length half that of open pipe i.e frequency remain same.

(4) At open ends in organ pipe antinode is slightly outside the pipe. So distance of antinode from open end is called end correction. Due to finite momentum of air particles the end correction was done by which



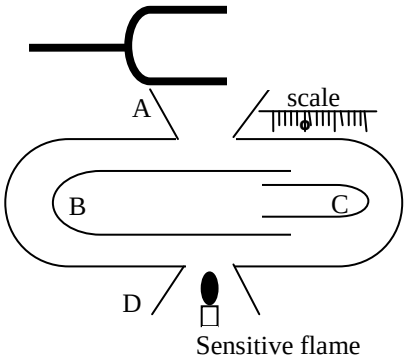
$$v_c = \frac{v}{4(L+0.6r)} \quad \text{and} \quad v_O = \frac{v}{2(L+1.2r)}$$

Where r is radius of pipe.

11. Apparatus for determining speed of sound

11.1 QUINCK'S TUBE :

It consists of two U shaped metal tubes. Sound waves with the help of tuning fork are produced at A which travel through B & C and comes out at D where a sensitive flame is present. Now the two waves coming through different paths interfere and flame flares up. But if they are not in phase, destructive interference occurs and flame remains undisturbed.



Suppose destructive interference occurs at D for some position of C. If now tube C is moved so that interference condition is disturbed and again by moving a distance x, destructive interference occurs so that

$$2x = \lambda$$

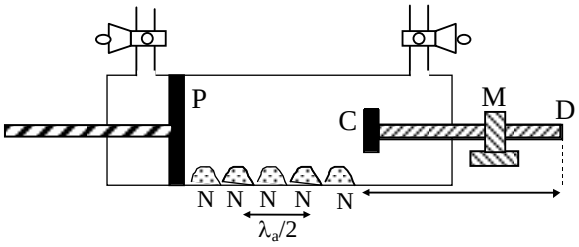
similarly if the distance moved between successive constructive and destructive interference is x then

$$2x = \frac{\lambda}{2}$$

Now by having value of x, speed of sound is given by

$$v = n\lambda$$

11.2 KUNDT'S TUBE : It is a tube used to determine speed of sound in different gases. It consists of a glass tube in which a small quantity of lycopodium powder is spread. The tube is rotated so that powder starts slipping. Now rod CD is rubbed at end D so that stationary waves form. The disc C vibrates so that air column also vibrates with the frequency of the rod. The piston P is adjusted so that frequency of air column become same as that of rod. So resonance occurs and column is thrown into stationary waves. The powder sets into oscillations at antinodes while heaps of powder are formed at nodes.



For rod $\ell_r = \frac{\lambda_r}{2}$

$$\lambda_r = 2\ell_r$$

and $v_r = n\lambda_r = n\,(2_r)$

For air $\lambda_a = \frac{\lambda_a}{2}$

$\lambda_a = 2_a$

and $v_a = n\lambda_a = n.(2\ell_a)$

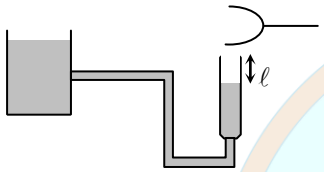
so $\frac{v_r}{v_a} = \frac{\ell_r}{\ell_a}$

$v_r = v_a.\frac{\ell_r}{\ell_a}$

11.3 RESONANCE TUBE :

It is a closed organ pipe having air column of variable length. If the frequency of a tuning fork is same as the frequency of vibration of air column in the tube then intensity of sound becomes maximum. This is said to be the condition of resonance.

The frequency is given by



$$\nu = \frac{(2n-1)v}{4\ell}$$

or $$\ell = \frac{(2n-1)v}{4v}$$

Here frequency is constant and length is variable. When water level in resonance tube is lowered, it gives resonance with same frequency at different length.

On neglecting the end correction, if ℓ is the minimum length of pipe for resonance, then other length for resonance are 3ℓ, 5ℓ, 7ℓ,.....

If we include the end correction, if ℓ is minimum length of pipe for resonance and x is end correction. Then other lengths of pipe for resonance are

$(\ell_1 + x) : (\ell_2 + x) : (\ell_3 + x)..... = 1 : 3 : 5.....$

12. Beats

When two sound waves of same amplitude travelling in same direction with different frequency superimpose, then intensity varies periodically with time. This effect is called Beats.

Consider two sound waves of frequency n_1 and n_2 propagating in the same direction so we have

$y_1 = a \sin(2\pi n_1 t - kx)$

$y_2 = a \sin(2\pi n_2 t - kx)$

where $n_1 - n_2 = \Delta n$

By principle of superposition

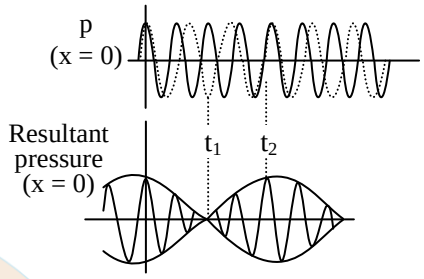
$y = y_1 + y_2$

$= a[\sin(2\pi n_1 t - kx) + \sin(2\pi n_2 t - kx)]$

$= a[\sin\{2\pi(n_2 + \Delta n)t - kx\} + \sin(2\pi n_2 t - kx)]$

But $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$y = [2a\cos(\Delta nt\pi)]\sin[2\pi n_2 t + \pi\Delta nt - kx]$



Thus frequency of variation of amplitude $= \frac{\Delta n}{2}$

$= \frac{|n_1 - n_2|}{2}$

We can explain the formation of beats graphically also. Ist figure shows the variation of pressure as a function of time at x = 0. At t = 0, two waves produce a change in pressure by which phase different grows. At time t_1 the two pressure changes becomes out of phase and the resultant becomes zero. At time t_2 the phase difference increases and hence the formation of beats occur.

frequency of variation of intensity

$\Delta n = |n_1 - n_2|$ = Beat frequency

12.1 BEAT TIME PERIOD :

Time interval between two successive maxima or minima is called Beat time period (T).

12.2 BEAT FREQUENCY :

Number of beats per second is called Beat frequency. If frequency of parent waves are n_1 and n_2, then

Beat frequency $= |n_1 - n_2|$

Beats time period $= \frac{1}{|n_1 - n_2|}$

Note :

(i) The frequency $|n_1 - n_2|$ should not be very large otherwise beats are not audible. It should be less than 16 Hertz.

(ii) Determination of unknown frequency with the help of beats.

(a) If tuning fork of unknown frequency (n_2) oscillates with tuning fork of known frequency (n_1) producing n beats then $n_2 = n_1 n$

(b) If wax is applied to arm of tuning fork of unknown frequency. Then its frequency decreases. Hence it oscillate with tuning fork of unknown frequency if

(i) Beat frequency increases then $n_2 = n_1 - n$

(ii) Beat frequency decreases then $n_2 = n_1 + n$

(c) If arm of tuning fork of unknown frequency is filed then its frequency increases. Hence it oscillates with tuning fork of known frequency if

(i) Beat frequency increases then $n_2 = n_1 + n$

(ii) Beat frequency decreases then $n_2 = n_1 - n$

(d) If n tuning fork are arranged in series that each tuning fork produces N beats with the previous one and frequency of last tuning fork is P times of first tuning fork then frequency of first tuning fork. = $\frac{(n-1)N}{P-1}$

BRAINS EDUCATION

SOLVED EXAMPLES

Ex.1 A wave of wavelength 0.60 cm is produced in air and it travels at a speed of 300 m/s. Will it be audible.

Sol. From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300\,m/s}{0.60\times10^{-2}\,m} = 50000 \text{ Hz.}$$

This is much above the audible range. It is an ultrasonic wave and will not be audible.

Ex.2 A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 1.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air 1.4×10^5 N/m^2.

Sol. The pressure amplitude is

$$p_0 = \frac{1.0\times10^{-3}\,N/m^2}{2} = 0.5 \times 10^{-3} \text{ N/m}^2.$$

The displacement amplitude s_0 is given by

$$p_0 = Bks_0$$

$$\text{or, } s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\pi B}$$

$$= \frac{0.5\times10^{-3}\,N/m^2\times(40\times10^{-2}\,m)}{2\times3.14\times1.4\times10^5\,N/m^2}$$

$$= 2.2 \times 10^{-10} \text{ m.}$$

Ex.3 The faintest sound, the human ear can detect at a frequency of 1 kHz (for which ear is most sensitive) correspond to an intensity of about 10^{-12} W/m^2 (the so called threshold of hearing). Determine the pressure amplitude and maximum displacement associated with this sound assuming the density of air = 1.3 kg/m^3 and velocity of sound in air = 332 m/s

Sol. As $I = (p_0^2/2\rho v)$; $p_0 = \sqrt{I\times2\rho v}$

$$\text{So } p_0 = \sqrt{(10^{-12})\times2\times1.3\times332}$$

$$= 2.94 \times 10^{-5} \text{ N/m}^2$$

Now as $p_0 = \rho v\omega A$

$$A = \frac{p_0}{\rho v\omega} = \frac{2.94\times10^{-15}}{1.3\times332\times(2\pi\times10^3)} = 1.1\times10^{-11} \text{ m}$$

From this it is clear that ear is extremely sensitive detector of sound waves as it can respond to amplitude about (1/10)th the size of an atom!

Ex.4 The wavelength of the note emitted by a tuning fork, frequency 512 Hz in air at 17ºC is 66.6 cm. If the density of air at S.T.P. is 1.293 gm per litre, calculate γ for air.

Sol. Given that n = 512 Hz and λ = 66.6 cm

Using $v = n\lambda$, we get

$v = 512 \times 66.6 = 340.48$ m/sec at 17ºC.

$$\text{Now } v_0 = \frac{vt}{1+t/546} = \frac{340.48}{1+17/546}$$

$$= 330 \text{ m/sec}$$

$$= 3.3 \times 10^4 \text{ cm/sec}$$

$$\text{We know that } v = \sqrt{\frac{\gamma P}{d}}$$

$$\text{or } \gamma = \frac{v^2 d}{P}$$

Here $P = 1.013 \times 10^6$ dynes/cm^2,

$d = 1.293 \times 10^{-3}$ gm/cm^3.

$$\therefore \quad \gamma = \frac{(3.3\times10^4)^2\times1.293\times10^{-3}}{1.013\times10^6} = 1.39$$

Ex.5 Determine the change in volume of 6 litres of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/cc and g = 9.81m/s^2]

Sol. For propagation of sound in liquid

$$v = \sqrt{B/\rho} \text{ i.e., } B = v^2\rho$$

But by definition $B = -V(\Delta P/\Delta V)$

$$\text{So } -V\frac{\Delta P}{\Delta V} = v^2\rho, \text{ i.e.} \Delta V = \frac{V(-\Delta P)}{\rho v^2}$$

Here $\Delta P = h_2\rho g - h_1\rho g$

$$= (75 - 200) \times 13.6 \times 981$$

$$= -1.667 \times 10^6 \text{ dynes/cm}^2$$

$$\text{So } \Delta V = \frac{(6\times10^3)(1.667\times10^6)}{0.81\times(1.280\times10^5)^2} = 0.75 \text{ cc}$$

Ex.6 Find the molecular weight for a gas in which the velocity of sound is 1260 ms^{-1} at 0ºC and whose γ is 1.4.

Sol. $C = \sqrt{\frac{\gamma P}{\rho}}$

$\therefore \quad 1260 = \sqrt{\frac{1.4P}{\rho}}$

$\Rightarrow \quad \frac{P}{\rho}$ of the gas at 0°C = 11.34×10^5

$P = \frac{\rho}{M}RT$ is the perfect gas equation in terms of density

$\Rightarrow \quad \frac{P}{\rho} = \frac{RT}{M}$

$\therefore \quad M = \frac{8.3 \times 273}{11.34 \times 10^5} = 2 \times 10^{-3}$ kg = 2g

Ex.7 The speed of sound in hydrogen is 1270 m/s. Calculate the speed of sound in a mixture of oxygen and hydrogen mixed in a volume ratio 1 : 4.

Sol. Suppose, in the mixture the volumes of oxygen and hydrogen are V_1 and V_2 respectively. If the densities are d_1 and d_2 respectively than their masses will be V_1d_1 and V_2d_2 respectively. If the density of the mixture is *d*, then

$$d = \frac{\text{Total mass}}{\text{Total volume}} = \frac{V_1d_1 + V_2d_2}{V_1 + V_2}$$

$$= \frac{V_2d_2\left(\frac{V_1d_1}{V_2d_2} + 1\right)}{V_2\left(\frac{V_1}{V_2} + 1\right)} = \frac{d_2\left(\frac{V_1d_1}{V_2d_2} + 1\right)}{\left(\frac{V_1}{V_2} + 1\right)}$$

Given : $\frac{V_1}{V_2} = \frac{1}{4}$,

$$\frac{d_1}{d_2} = \frac{\text{Molecular weight of oxygen}}{\text{Molecular weight of hydrogen}}$$

$$= \frac{32}{2} = \frac{16}{1}$$

$$\therefore \quad d = \frac{d_2\left(\frac{1}{4} \text{x} \frac{16}{1} + 1\right)}{\frac{1}{4} + 1} = 4d_2 \quad \text{or} \quad \frac{d}{d_2} = 4$$

Let v be the speed of sound in the mixture and v_2 in hydrogen. Then by Laplace's formula, we have

$$v = \sqrt{\left(\frac{\gamma P}{d}\right)} \text{ and } v_2 = \sqrt{\left(\frac{\gamma P}{d_2}\right)}$$

$$\therefore \quad \frac{v}{v_2} = \sqrt{\left(\frac{d}{d_2}\right)} = \sqrt{4} = 2$$

$$\therefore \quad v = \frac{v_2}{v} = \frac{1270}{2} = 635 \text{ m/s}$$

Ex.8 Find the amplitude of vibration of the particles of air through which a sound wave of intensity 2.0×10^{-6} W/m^2 and frequency 1.0 kHz is passing. Density of air = 1.2 kg/m^3 and speed of sound in air = 330 m/s.

Sol. The relation between the intensity of sound and the displacement amplitude is

$$I = 2\pi^2 s_0^2 \nu^2 \rho_0 v$$

$$\text{or} \quad s_0^2 = \frac{I}{2\pi^2 \nu^2 \rho_0 v}$$

$$= \frac{2 \times 10^{-6} \text{W/m}^2}{2\pi^2 \times (1.0 \times 10^6 \text{s}^{-2}) \times (1.2 \text{kg/m}^3) \times (330 \text{m/s})}$$

$= 2.53 \times 10^{-16}$ m^2

or $s_0 = 1.6 \times 10^{-8}$ m

Ex.9 What is the maximum possible sound level in dB of sound waves in air. Given that density of air = 1.3 kg/m^3, v = 332 m/s and atmospheric pressure P = 1.01×10^5 N/m^2.

Sol. For maximum possible sound intensity, pressure amplitude of wave will be equal to atmospheric pressure, i.e.,

$p_0 = P = 1.01 \times 10^5$ N/m^2

$$I = \frac{p_0^2}{2\rho v} = \frac{(1.01 \times 10^5)^2}{2 \times 1.3 \times 332} = 1.18 \times 10^7$$

$$\therefore \quad SL = 10\log \frac{I}{I_0} \approx 10\log \frac{10^7}{10^{-12}} = 190\text{dB}$$

Ex.10 In a large room a person receives direct sound waves from a source 120 m away form him. He also receives waves from the same source which reach him, being reflected from the 25 m high ceiling at point half way between them. For which wavelength will these two sound waves interfere constructively.

Sol. As shown in fig. for reflection from the ceiling

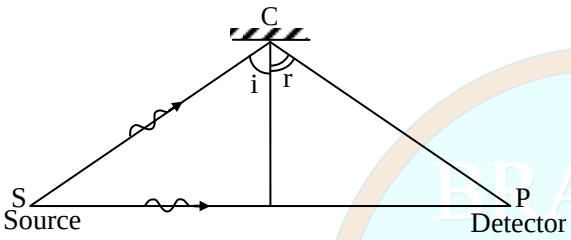
Path SCP = SC + CP = 2SC

[as $\angle i = \angle r$, SC = CP]

path SCP = $2\sqrt{60^2+25^2}$ = 130 m

So path difference between interfering waves along paths SCP and SP

$\Delta x = 130 - 120 = 10$ m



Now for constructive interference at P

$\Delta x = n\lambda$, i.e. $10 = n\lambda$

or $\lambda = \frac{10}{n}$ with n = 1,2,3.....

i.e. λ = 10m, 5m, (10/3) m and so on

Ex.11 Three plane sources of sound of frequency n_1 = 400 Hz, n_2 = 401 Hz and n_3 = 402 Hz of equal amplitude a each, are sounded together. A detector receives waves from all the three sources simultaneously. It can detect signals of amplitude ≥ a. Calculate.

(a) period of one complete cycle of intensity received by the detector and

(b) time for which the detector remains idle in each cycle of intensity.

Sol. (a) We know that $y = a \sin \omega t = a \sin (2\pi nt)$

The displacement of the medium particles caused by these waves are given as

$y_1 = a \sin (800\ \pi t)$ ....(1)

$y_2 = a \sin (802\ \pi t)$ ....(2)

and $y_3 = a \sin (804\ \pi t)$ ....(3)

The resultant displacement of medium particle at time *t* is given by

$y = y_1 + y_2 + y_3$

$= a [\sin(800\ \pi t) + \sin (802\ \pi t) + \sin (804\ \pi t)]$

$= a[\{\sin(800\ \pi t) + \sin (804\ \pi t)\} + \sin (802\ \pi t)]$

$= a (1 + \cos 2\ \pi t) \sin 802\ \pi t$

The resultant is also a plane wave. Let its amplitude be A. Then, $y = A \sin \omega t$.

$\therefore$ $A = a (1 + \cos 2\pi t)$ ....(4)

Equation (4) show that the resultant amplitude (or resultant intensity $I \propto A^2$) varies with time. The resultant amplitude is maximum, when

$2\pi t = 0, 2\pi, 4\pi, ...$

or $t = 0, 1, 2, 3, ...$

Hence period of one complete cycle of intensity is **one second.**

(b) Given that signal is detected when A ≥ a

$\therefore$ $\cos 2\ \pi\ t \geq 0$

Thus $\cos 2\pi\ t$ should lie either in first quadrant or in fourth quadrant i.e., between either 0 to $\pi/2$ or $3\pi/2$ and 2π. So during first cycle of intensity, the signal is detected when

$0 \leq t \leq \frac{1}{4}$ and $\frac{3}{4} \leq t \leq 1$

This shows that detector remains deal from

$t = \frac{1}{4}$ sec to $t = \frac{3}{4}$ sec.

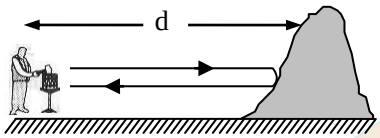
Therefore, in each cycle of intensity, the detector remains ideal for $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = 0.5$ sec

Ex.12 A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90m and find the echo is again not heard when the drumming rate becomes 60 per minute. Calculate - (a) the distance between the mountain and the initial position of the man (b) the velocity of sound.

Sol. Initially as the drumming rate is 40 per minute, i.e. time interval between two successive drum beats is (60/40), i.e. (3/2) s and as echo is not heard distinctly, time taken by sound to travel to hill and come back is (3/2) s. So if d is the distance between man and hill and v the velocity of sound –

$$\frac{d}{v}+\frac{d}{v}=\frac{2d}{v}=\frac{3}{2} \qquad ....(1)$$

When man moves 90m towards the hill, distance between him and hill will be $(d - 90)$ and as echo is not heard again at drumming rate of 60 per minute, i.e., for time interval (60/60) = 1 sec, so



$$\frac{2(d-90)}{v}=1 \qquad ....(2)$$

Solving equations (1) and (2) for d and v, we find (a) d = 270 m (b) v = 360 m/s

Ex.13 The length of a wire between the two ends of a sonometer is 105 cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15**.**

(A) 75 cm, 25 cm, (B) 25 cm, 75 cm,

(C) 75 cm, 100 cm, (D) None

Sol. **(C)**

From the law of length of stretched string, we have $n_1\ell_1 = n_2\ell_2 = n_3\ell_3$

Here $n_1 : n_2 : n_3 = 1 : 3 : 15$

$$\therefore \frac{\ell_1}{\ell_2}=\frac{n_1}{n_2}=\frac{3}{1} \text{ and } \frac{\ell_1}{\ell_3}=\frac{n_3}{n_1}=15/1$$

$$\ell_2=\frac{\ell_1}{3} \text{ and } \ell_3=\frac{\ell_1}{15}$$

The total length of the wire is 105 cm.

Therefore $\ell_1 + \ell_2 + \ell_3 = 105$

$$\text{or } \ell_1+\frac{\ell_1}{3}+\frac{\ell_1}{15}=105 \quad \text{or} \quad \frac{21\ell_1}{15}=105$$

$$\ell_1=\frac{105\times15}{21}=75 \text{ cm.}$$

$$\therefore \ell_2=\frac{\ell_1}{3}=\frac{75}{3}=25 \text{ cm}$$

$$\ell_3=\frac{\ell_1}{15}=\frac{75}{15}=5 \text{ cm}$$

Hence the bridge should be placed at 75 cm and (75 + 25) = 100 cm from one end.

Ex.14 A closed pipe and an open pipe sounding together produce 5 beats per second. If the length of the open pipe is 30 cm, find by how much the length of the closed pipe must be changed to bring the two pipe in unison –

(A) 14.86 cm (B) 15.14 cm

(C) 15 cm (D) 0.14 cm

Sol. **(D)**

Frequency of the open pipe,

$$n=\frac{v}{2\ell}=\frac{33000 \text{ cm/s}}{2\times30} \Rightarrow n = 550 \text{ Hz,}$$

Since this pipe produce 5 beats per sec. with the closed pipe, the frequency of the closed pipe is 550 ± 5 = 555 or 545 Hz. If the length of the closed pipe is, then its fundamental frequency is v/4.so

$$v/4\ell = 555 \text{ or } 545$$

$$\Rightarrow \frac{33000}{4\ell}=555 \text{ or } 545$$

$$\Rightarrow \ell = 14.86 \text{ or } 15.14 \text{ cm}$$

For unison, the frequency of the closed pipe must also be 550, suppose, for this its length is to be changed by x cm. Then

$$\Rightarrow \frac{33000}{4(\ell\pm x)}=550$$

$$\Rightarrow \ell \pm x = 15 \text{ cm}$$

$$\Rightarrow 14.86 + x = 15$$

$$\text{or } 15.14 - x = 15$$

$$\therefore x = 0.14 \text{ cm.}$$

Ex.15 An under water swimmer sends a sound signal to the surface. If it produces 5 beats per second when compared with the fundamental tone of a pipe of 20 cm length closed at one end, what is the wavelength of sound in water (Take v_{water} = 1500 m/s and v_{air} = 360 m/s)

Sol. If f is the frequency of sound (which is independent of medium), according to given problem $f = f_C \pm 5$

But as $f_C = \frac{v}{4L} = \frac{360}{4\times 0.2} = 450$ Hz

So $f = (450 \pm 5)$ Hz

Now as $v = f\lambda$, i.e. $\lambda = v/f$

So $\lambda_w = \frac{v_w}{f_w} = \frac{v_w}{f}$ [as $f_w = f_A = f$]

i.e. $\lambda_w = \frac{1500}{450 \pm 5}$

i.e. $\lambda_w = 3.30$ m or 3.37 m

Ex.16 An open organ pipe has a fundamental frequency of 300 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? The velocity of sound in air = 350 ms^{-1}.

Sol. For a closed pipe $n = \frac{c}{4\ell}$ and 3n, 5n, 7n,.....are the overtones. For an open pipe $n = \frac{c}{2\ell}$ and 2n, 3n, 4n,.....are the overtones.

$\Rightarrow \ell = \frac{c}{2n} = \frac{350}{2\times 300} = 0.58$ m

The frequency of the first overtone

$= 2n = 2 \times 300 = 600$ Hz

$\therefore$ The frequency of the first overtone of the closed pipe = 600 = 3n

n = 200Hz

$\therefore \quad 200 = \frac{350}{4\ell}$ or $\ell = \frac{350}{4\times 200} = 0.44$ m

Ex.17 A column of air and a tuning fork produce 4 beats per second when sounded together. The tuning fork gives the lower note. The temperature of air is 15ºC. When the temperature falls to 10ºC, the two produce 3 beats per second. Find the frequency of the fork.

Sol. Let the frequency of the tuning fork be n Hertz. Then

frequency of air column at 15ºC = n + 4

and frequency of air column at 10ºC = n + 3

According to $v = n\lambda$, we have

$v_{15} = (n + 4)\lambda$ and $v_{10} = (n + 3)\lambda$.

$\therefore \quad \frac{v_{15}}{v_{10}} = \frac{n+4}{n+3}$

The speed of sound is directly proportional to the square–root of the absolute temperature.

$\therefore \quad \frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}} = \sqrt{\frac{288}{283}}.$

$\therefore \quad \frac{n+4}{n+3} = \sqrt{\frac{288}{283}} = \left(1+\frac{5}{283}\right)^{1/2}$

or $\quad 1 + \frac{1}{n+3} = 1 + \frac{1}{2}\times\frac{5}{283} = 1 + \frac{5}{566}$

or $\quad \frac{1}{n+3} = \frac{5}{566}$ or $n + 3 = \frac{566}{5} = 113$

$\therefore \quad n =$ **110 Hz.**

Ex.18 An aluminium rod 80 cm long clamped at the middle is used to create standing waves in a Kundt's tube experiment. If the tube is 1 meter long and the dust heaps accumulate at 6 cm, intervals and velocity of sound in air is 330 m/s. then determine the velocity of sound in the rod.

Sol. For Kundt's tube :

$\frac{V_{rod}}{V_{air}} = \frac{\ell_{rod}}{\ell_{air}} = \frac{80}{6}$

Thus $V_{rod} = \frac{80}{6} \times 330 = 4400$ m/s.

Ex.19 In the Quincke's interference tube as shown in the theory, it is found that the sound intensity has a minimum value of 100 units in one position of tube C and continuously climbs to a maximum value of 900 units at a second position 1.70 cm from the first. If the velocity of the sound is 340 m/s, then determine the frequency of the sound emitted from the source.

Sol. It is given that for consecutive destructive and constructive interference, the tube C has to be displaced by x = 1.70 cm. Therefore

$\frac{\lambda}{2} = 2x$ or $\lambda = 4 \times 1.70 = 6.8$ cm

The frequency

$n = \frac{V}{\lambda} = \frac{340}{6.8\times 10^{-2}} = 5000$ Hz